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1994 J. Phys. A: Math. Gen. 27 L219

(http://iopscience.iop.org/0305-4470/27/7/005)

J. Phys. A: Math. Gen. 27 (1994) L219–L221. Printed in the UK

LETTER TO THE EDITOR

The anomalous quantum Hall current driven by a non-uniform magnetic field

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Received 27 October 1993, in final form 4 January 1994

Abstract. The existence of an electric current driven by the gradient of the magnetic field in an idealized quantum Hall effect device is predicted as a consequence of the anomalous magnetic moment of the electron ($g - 2 \neq 0$) along with the exact Landau ground-state degeneracy for the two-dimensional Pauli Hamiltonian with an arbitrary magnetic field.

We begin with the simplest model of independent two-dimensional electrons with spins polarized in the z direction by a magnetic field $B(x, y) \geqslant 0$ as described by the two-dimensional Pauli Hamiltonian

$$H_0 = \frac{1}{2m}\left[\left(-i\hbar\frac{\partial}{\partial x} - \frac{e}{c}A_x(x, y)\right)^2 + \left(-i\hbar\frac{\partial}{\partial y} - \frac{e}{c}A_y(x, y)\right)^2\right] - \frac{e\hbar}{2mc}B(x, y) \qquad (1)$$

with

$$B(x, y) = \frac{\partial}{\partial x}A_y(x, y) - \frac{\partial}{\partial y}A_x(x, y).$$

Aharonov and Casher [1] have proved that H_0 possesses a highly degenerate lowest Landau level (LLL) with a density of states equal to

$$n(B) = \frac{eB(x, y)}{hc} \qquad (2)$$

and which consists of the wavefunctions satisfying

$$\left[\hbar\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right) + i\frac{e}{c}(A_x - iA_y)\right]\phi = 0. \qquad (3)$$

In particular the LLL can be spanned by 'generalized coherent states' [2] localized around points (x, y). It follows that, for the electrons in the LLL, the x–y plane becomes a phase space of a certain fictitious dynamical system with one degree of freedom [2] (see [3,4]

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for a uniform magnetic-field case). Adding an external potential $V(x, y)$, we obtain a Hamiltonian

$$H = H_0 + V. \tag{4}$$

If V is small and the temperature is low, the Landau levels mixing can be neglected [5] and, as shown in [2] (see also [6, 7]), the dynamics of the electron in the LLL can be described by the following action functional on the phase space $\mathbb{R}^2$

$$S = \int \left[\frac{e}{c} A_k(\xi)\dot{\xi}^k - V(\xi)\right] \mathrm{d}t \tag{5}$$

where $k = 1, 2$, $\xi^1 = x$, $\xi^2 = y$. The corresponding Hamiltonian equations read

$$\frac{e}{c} F_{k\ell}(\xi)\dot{\xi}^\ell = \partial_k V(\xi) \tag{6}$$

where $F_{k\ell} = \partial_k A_\ell - \partial_\ell A_k$. Using (2) and (6) the following semiclassical expression for the electric current flowing through a curve with end points ξ', ξ'' has been obtained [2]

$$\begin{aligned} j &= -e \int_c n(B)\epsilon_{k\ell}\dot{\xi}^\ell \,\mathrm{d}\xi^k \\ &= -\frac{e^2}{hc}\int B\epsilon_{k\ell}\dot{\xi}^\ell \,\mathrm{d}\xi^k = -\frac{e}{hc}\left[V(\xi'') - V(\xi')\right]. \end{aligned} \tag{7}$$

Putting $V(\xi) = -eU(\xi)$, where U is an external electric potential, we recover the basic formula for the quantum Hall current (filling factor = 1) [5]. However, in the case of a non-uniform magnetic field, the Zeeman term in the Hamiltonian is important and its realistic form is given by

$$H_{\mathrm{Zeeman}} = \frac{ge\hbar}{4mc} B(x, y) \tag{8}$$

where generally $g \neq 2$. (In an ideal system m and g have their vacuum values; otherwise they are taken as effective parameters.) Hence, the external potential which should be added to the Pauli Hamiltonian (1) is

$$V(x, y) = -eU(x, y) - (g - 2)\frac{e\hbar}{4mc} B(x, y). \tag{9}$$

It follows that, for the sample in the form of a strip whose long edges coincide with lines of a constant magnetic field B_1 and B_2, the formula for the Hall voltage V_{H} should be the following

$$V_{\mathrm{H}} = R_{\mathrm{H}} J + (g - 2)\frac{\hbar}{4mc}(B_2 - B_1) \tag{10}$$

with $R_{\mathrm{H}} = h/e^2$. Hence even for $V_{\mathrm{H}} = 0$ we have an anomalous current $J_e = (g - 2)(e^2/8\pi mc)(B_2 - B_1)$ flowing in the sample.

One should mention that a similar correction to the quantum Hall current in a non-uniform magnetic field was recently obtained by Frölich and Studer [8, 9] using a completely different approach. They applied linear response theory to a phenomenological Chern–Simons Lagrangian and their correction is roughly proportional to g and not to $(g - 2)$ like ours. This is due to the fact that the degeneracy of the LLL for the Pauli Hamiltonian was not taken into account in their formalism.

We thank J Lewandowski, A MacDonald, M Meisel, J R Schrieffer and U M Studer for helpful discussions. One of the authors (RA) is grateful to the Kosciuszko Foundation for a Research Scholarship and to the Department of Physics, University of Florida, for hospitality. Partial support from KBN (project PB 1436/2/91) is also acknowledged.

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